Wine Selection: Tasting, Learning and Identification of Favorites

Francisco Blasques[1]
Maastricht University.

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Abstract

This paper lays down sufficient conditions for the appropriate convergence of wine consumer preferences as a result of repeated wine tasting events. Failure to satisfy these conditions might result in the inability of the sequence of preferred wines to converge to an appropriate well defined point, even in the limit, in the event of infinitely many wine tasting experiences. This fact could have important practical implications.

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[1]Corresponding author: f.blasques@maastrichtuniversity.nl., Department of Quantitative Economics, Maastricht University, P.O. Box 616, 6200 MD Maastricht, The Netherlands. The author is thankful to Rita Silva, Eric Beutner and Marco Avarucci for helpful comments and suggestions. The usual disclaimer applies.

1 Introduction

Central to the result of this paper are the characteristics of any given wine. Let $\Theta \subseteq \mathbb{R}^k$ denote the *wine space*, i.e. the space of all possible wines, as defined by their distinguishable characteristics, along k dimensions. Each element of Θ, a k-dimensional vector $\boldsymbol{\theta}$, is thus a full characterization of any given wine. Given any pair of wines $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \in \Theta \times \Theta$, the statement $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$ implies that both wines are *indistinguishable* from one another at the eyes of the consumer. On the contrary, $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$ implies that these two wines are *distinguishable*, at least along one of the k dimensions that characterize each of the two wines. It is thus important to stress that $\boldsymbol{\theta} \in \Theta$ is potentially a very large dimensional vector that contains all the *distinguishable information* about the wine tasting experience. Wine comes in various degrees of sweetness, acidity and tannins, it exhibits different properties in terms of balance and body, it is consumed in different occasions, in different seasons of the year, it is served at different temperatures, and accompanying different meals. All these properties and many more, are what makes a given wine distinct from others. Hence, note that if $\boldsymbol{\theta}_1 \in \Theta$ denotes a portuguese red 1999 *Barca Velha*, served at 16°C on a cold December night, accompanied by a stew of marinated Hare, $\boldsymbol{\theta}_2 \in \Theta$ might still denote the same 1999 *Barca Velha* but this time drank at a possibly different temperature, or accompanied by a different meal on a warm spring afternoon.

In the following sections we use the *wine space* Θ as a convenient framework to obtain the desired result on convergence of consumers' preferences to unique favorite wines. Section 2 introduces the preference system of the wine consumer, discusses convenient representations of the preference system and defines the way preferences change in time, in an appropriate stochastic environment. In section 3 the main result of this paper is stated in the form of a theorem. The interpretation of the result and the nature of its assumptions are briefly discussed. Finally, section 4 discusses the practical relevance of the present paper and concludes.

2 Preferences

Different consumers have different opinions over the elements of Θ in terms of which are preferred and which are less appreciated. The preferences of any given wine consumer, over all existing wines $\boldsymbol{\theta} \in \Theta$, are defined by a *wine preference relation* $\precsim$, i.e. a complete preordering of the elements of Θ.

Definition 1. (Wine Preference Relation) *A wine preference relation is a complete preordering on the wine space* Θ*, i.e. it is a binary relation* $\precsim$ *defined for every* *pair of wines* $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \in \Theta \times \Theta$ *satisfying (i) reflexivity* $\boldsymbol{\theta} \precsim \boldsymbol{\theta}\, \forall \boldsymbol{\theta} \in \Theta$ *and (ii) transitivity* $\boldsymbol{\theta}_1 \precsim \boldsymbol{\theta}_2 \wedge \boldsymbol{\theta}_2 \precsim \boldsymbol{\theta}_3 \Rightarrow \boldsymbol{\theta}_1 \precsim \boldsymbol{\theta}_3\, \forall (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3) \in \Theta \times \Theta \times \Theta$.

Under appropriate conditions, the *wine preference relation* $\precsim^i$, of consumer i, defined on the *wine space* Θ, can be conveniently characterized by a *wine preference mapping* $Q^i : \Theta \to \mathbb{R}$. For every $\boldsymbol{\theta} \in \Theta$, $Q^i(\boldsymbol{\theta}) \in \mathbb{R}$ is well understood as the "value" or the "ranking" that consumer i attributes to wine $\boldsymbol{\theta} \in \Theta$.

Lemma 1. (Wine Preference Mapping) *Let* Θ *be a connected subset of* $\mathbb{R}^k$ *and suppose that the wine preference relation* $\precsim^i$ *is such that for every* $\boldsymbol{\theta}' \in \Theta$, *the sets* $\underline{\Theta} = \{\boldsymbol{\theta} \in \Theta : \boldsymbol{\theta} \precsim^i \boldsymbol{\theta}'\}$ *and* $\bar{\Theta} = \{\boldsymbol{\theta} \in \Theta : \boldsymbol{\theta}' \precsim^i \boldsymbol{\theta}\}$ *are closed in* Θ. *Then, there exists a preference mapping* $Q^i : \Theta \to \mathbb{R}$ *that completely characterizes* $\precsim_i$, *satisfying* $Q^i(\boldsymbol{\theta}_1) = Q^i(\boldsymbol{\theta}_2)$ *if* $\boldsymbol{\theta}_1 \sim^i \boldsymbol{\theta}_2$, *and* $Q^i(\boldsymbol{\theta}_1) < Q^i(\boldsymbol{\theta}_2)$ *if* $\boldsymbol{\theta}_1 \prec^i \boldsymbol{\theta}_2$, *for every pair* $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \in \Theta \times \Theta$, *where* $\boldsymbol{\theta}_1 \sim^i \boldsymbol{\theta}_2 \equiv \{\boldsymbol{\theta}_1 \precsim^i \boldsymbol{\theta}_2 \text{ and } \boldsymbol{\theta}_2 \precsim^i \boldsymbol{\theta}_1\}$ *while* $\boldsymbol{\theta}_1 \prec^i \boldsymbol{\theta}_2 \equiv \{\boldsymbol{\theta}_1 \precsim^i \boldsymbol{\theta}_2 \text{ and not } \boldsymbol{\theta}_2 \precsim^i \boldsymbol{\theta}_1\}$.

Proof. See e.g. Debreu (1959). □

Let us now introduce stochastic time variation of wine preferences. We start by noting that a wine consumer is not "born" with a wine preference relation $\precsim^i$ that remains unchanged over time. On the contrary, opinions about wines evolve over time according to information acquired through wine tasting experiences. We thus let the preference relation be indexed by time $T \in \mathbb{N}$, thus denoting it $\precsim^i_T$. Furthermore, since $\precsim^i_T$ changes according to the particular history of wine tasting experiences of consumer i, we let $\precsim^i_T$ be a mapping $\precsim^i_T : \Omega \to \Phi$, where Ω denotes the space of all possible histories of wine tasting experiences, and Φ the space of all wine preference relations. Hence, $\precsim^i_T (\omega) \in \Phi$ denotes the wine preference relation of consumer i, at time T, given that the history $\omega \in \Omega$ of wine tasting experiences occurs. Likewise, $Q^i_T : \boldsymbol{\Theta} \times \Omega \to \mathbb{R}$ denotes the wine preference mapping of this consumer at time T. Finally, we let wine tasting histories occur according to a probabilistic setting, by taking Ω to be the event space of a complete probability space $(\Omega, \mathcal{F}, P)$, where P denotes the probability measure defined on $\mathcal{F}$, a σ-algebra generated by the measurable sets of Ω.

We are finally ready to proceed with the central task of this paper: *to provide conditions for the sequence of favorite wines* $\boldsymbol{\theta}^i_T := \arg\max_{\boldsymbol{\theta} \in \Theta} Q^i_T(\boldsymbol{\theta})\, \forall T \in \mathbb{N}$ *to converge to a given wine* $\boldsymbol{\theta}^i_\infty$, *in the limit, as the number of wine tasting experiences accumulate to infinity.* We thus search for conditions which ensure that $\boldsymbol{\theta}^i_T \to \boldsymbol{\theta}^i_\infty$ almost surely (a.s.) as $T \to \infty$, i.e. $P(\omega \in \Omega : \lim_{T\to\infty} |\boldsymbol{\theta}^i_T - \boldsymbol{\theta}^i_\infty| \geq \epsilon) = 0\, \forall \epsilon > 0$, where $\boldsymbol{\theta}^i_\infty := \arg\max_{\boldsymbol{\theta} \in \Theta} Q^i_\infty(\boldsymbol{\theta})$ and Q^i_∞ is some deterministic function to which Q^i_T converges in some appropriate sense. As we shall now see, these conditions are sometimes counterintuitive.

3 A Favorite Wine Theorem

Let us observe the main result first and discuss the nature of the assumptions involved later.

Theorem 1. *Let the asymptotic wine preference mapping* $Q^i_\infty : \Theta \to \mathbb{R}$ *be continuous on the wine space* Θ*, a compact subset of* $\mathbb{R}^k$*,* $k \in \mathbb{N}$*, and let it exhibit a unique maximum, a favorite wine* $\boldsymbol{\theta}^i_\infty = \arg\max_{\boldsymbol{\theta}\in\Theta} Q^i_\infty(\boldsymbol{\theta})$*. Then, if the wine preference mapping* Q^i_T *converges asymptotically to* Q^i_∞ *for almost every wine tasting history* $\omega \in \Omega$ *and uniformly in the wine space* Θ*, i.e. if* $P(\omega \in \Omega : \lim_{T\to\infty} \sup_{\boldsymbol{\theta}\in\Theta} |Q^i_T(\boldsymbol{\theta}) - Q^i_\infty(\boldsymbol{\theta})| \geq \epsilon) = 0 \,\forall \epsilon > 0$*, we have that as* $T \to \infty$ *the sequence of preferred wines* $\boldsymbol{\theta}^i_T$ *converges to* $\boldsymbol{\theta}^i_\infty$ *for almost every wine tasting history* $\omega \in \Omega$*.*

Proof. This proof follows standard consistency proofs for consistency of M-estimators (see e.g. Gallant and White (1988, ch.3) and P¨otscher and Prucha (1997, ch.3)). Let $\delta(\boldsymbol{\theta}^i_\infty)$ denote an open set in $\mathbb{R}^k$ containing $\boldsymbol{\theta}^i_\infty$. Then $\bar{\delta}(\boldsymbol{\theta}^i_\infty) \cap \Theta$ where $\bar{\delta}(\boldsymbol{\theta}^i_\infty)$ is the complement of $\delta(\boldsymbol{\theta}^i_\infty)$ in $\mathbb{R}^k$ is compact. Hence $\max_{\boldsymbol{\theta}\in\bar{\delta}(\boldsymbol{\theta}^i_\infty)\cup\Theta} Q^i_\infty(\boldsymbol{\theta})$ exists by Weierstrass's theorem. Denote, $\epsilon = Q^i_\infty(\boldsymbol{\theta}^i_\infty) - \max_{\boldsymbol{\theta}\in\delta(\bar{\boldsymbol{\theta}}^i_\infty)\cup\Theta} Q^i_\infty(\boldsymbol{\theta})$. Let A^i_T be the event $|Q^i_T(\boldsymbol{\theta}) - Q^i_\infty(\boldsymbol{\theta})| < \epsilon/2 \,\forall \boldsymbol{\theta}$. Then, $A^i_T \Rightarrow Q^i_\infty(\boldsymbol{\theta}^i_T) > Q^i_T(\boldsymbol{\theta}^i_T) - \epsilon/2$ and $A^i_T \Rightarrow Q^i_T(\boldsymbol{\theta}^i_\infty) > Q^i_\infty(\boldsymbol{\theta}^i_\infty) - \epsilon/2$. But because $Q^i_T(\boldsymbol{\theta}^i_T) \geq Q^i_T(\boldsymbol{\theta}^i_\infty)$ by the definition of $\boldsymbol{\theta}^i_T$, we have that $A^i_T \Rightarrow Q^i_\infty(\boldsymbol{\theta}^i_T) > Q^i_T(\boldsymbol{\theta}^i_\infty) - \epsilon/2$. Together, these conditions imply that $A^i_T \Rightarrow Q(\boldsymbol{\theta}_T{}^i) > Q^i_\infty(\theta^i_\infty) - \epsilon$ and hence that $A^i_T \Rightarrow \boldsymbol{\theta}^i_T \in \delta(\boldsymbol{\theta}^i_\infty)$. This implies $P(A_T) \leq P(\boldsymbol{\theta}^i_T \in \delta(\boldsymbol{\theta}^i_\infty))$ and hence we have $\boldsymbol{\theta}^i_T \overset{a.s.}{\to} \boldsymbol{\theta}^i_\infty$ because $P(\lim_{T\to\infty} A^i_T) = 1$. □

Curiously enough, to obtain $\boldsymbol{\theta}^i_\infty$ as the unique favorite wine in the limit as $T \to \infty$, we need more than just having a single peaked Q^i_∞ at $\boldsymbol{\theta}^i_\infty$ and to establish the convergence of $Q^i_T(\boldsymbol{\theta}) \to Q^i_\infty(\boldsymbol{\theta})$ a.s. as $T \to \infty$ for every wine $\boldsymbol{\theta} \in \Theta$. Indeed, a host of other conditions have been used in Theorem 1. Several interesting features of this theorem are thus worth commenting.

Note first that the assumption of a compact wine space Θ implies that for every wine characteristic (for every dimension of Θ) there exists an element $\boldsymbol{\theta} \in \Theta$ which attains the maximum value and one which attains a minimum value in that dimension. It is indeed easy to think of examples where the failure to comply with this assumption results in a sequence of $\boldsymbol{\theta}^i_T$'s that no longer converges to $\boldsymbol{\theta}^i_\infty$.

The continuity of the limit wine preference map Q^i_∞ implies that for arbitrary small changes in the properties of any given wine must correspond an arbitrarily small change in the "value" or "ranking" that consumer i attributes to that wine.

Once more, it is easy to devise examples where the failure to comply with this condition may lead to the failure of the result established in Theorem 1.

Finally, note that the a.s. uniform convergence of the sequence of wine preference maps $\{Q_T^i\}_{T=1}^{\infty}$ to the limit continuous preference Q_∞^i as $T \to \infty$ imposes a minimum rate of convergence of Q_T^i to Q_∞^i uniformly over Θ. Indeed, there cannot exist a sequence of wines $\boldsymbol{\theta} \in \Theta$ whose preference values converge at a speed that is arbitrarily close to zero. Necessary and sufficient conditions for the uniform convergence on the compact metric space Θ, $\sup_{\theta\in\Theta} |Q_T^i(\cdot;\boldsymbol{\theta}) - Q_\infty^i(\boldsymbol{\theta})| \to 0$ a.s. on Θ are that *(i)* $Q_T^i(\boldsymbol{\theta}) - Q_\infty^i(\boldsymbol{\theta}) \to 0$ a.s. for every $\boldsymbol{\theta} \in \Theta$ and *(ii)* that $\{Q_T^i(\cdot,\boldsymbol{\theta}), T \in \mathbb{N}\}$ be strongly asymptotically uniformly stochastically equicontinuous (see e.g. Newey (1991) and Davidson (1994, p.337)). Well known illustrative examples have been devised that illustrate the crucial role that the uniform convergence of Q_T^i plays in guaranteeing that $\boldsymbol{\theta}_T^i \to \boldsymbol{\theta}_\infty^i$ a.s. as $T \to \infty$.

4 Final Remarks

The concept of *preferred wine* plays an important role in both consumer's choice and producer's decision theory for the wine industry. Wine competitions, for instance, are centered around the idea of obtaining a ranking of wines that is derived from individual consumer preferences. On the consumer's side, the concept of choosing a *preferred wine* among a collection of alternatives relies on the ability of consumers to appropriately shape their preferences according to their wine tasting experiences. On the production side, the standard business belief that an increase in the overall "quality" of wine production might somehow "attract" more consumers and thus "increase demand", is centered in the idea that by repeated wine tasting events, the "higher quality" of the wine is somehow assimilated by consumers, which are eventually able to identify the wines of their choice. The present paper gives proper foundations to the concepts, ideas and beliefs just mentioned.

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